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WAVE EXCITATION IN A TWO-LAYERED MEDIUM BY AN OSCILLATING STAMP

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The antiplane problem of wave excitation in a two-layered medium consisting of an elastic layer and a rigidly connected half-space, set in oscillation by a stamp, is considered. On the basis of the physical principle of ultimate absorption [1, 2], the problem of the oscillation of a source on a surface is solved and consequently, the integral equation of a mixed problem is derived. A detailed study of the dispersion equation by using an electronic digital computer preceded the solution of the problem. The solution of the contact problem is constructed and a numerical analysis is made of the solutions obtained for specific values of the parameters.

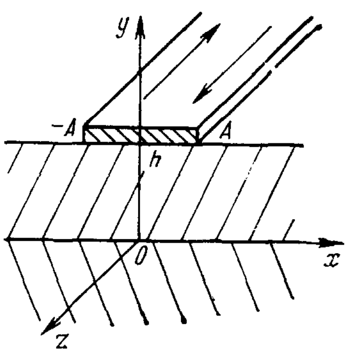

Fig. 1

1. The case is considered when forces $\tau_{xy} = \operatorname{Re}[\tau(x)e^{-i\omega t}]$ independent of the z coordinate are applied to a surface layer of thickness h in the domain $X \in [-A, A]$ (Fig. 1), and there are no normal stresses. The oscillations are assumed steady-state.

Then applying the principle of ultimate absorption by the Fourier transform method, we arrive at the following formulas describing the displacements $w(x, y, t)$ and $w_1(x, y, t)$ for the layer and the half-space, respectively:

$$w(x, y, t) = \operatorname{Re}[W_1(x, y)e^{-i\omega t}] \tag{1.1}$$

$$W(x,y) = \frac{1}{2\pi}\int_{-a}^{a}\int_{\Gamma}\frac{(\sigma+G\sigma_1)e^{\sigma y}+(\sigma-G\sigma_1)e^{-\sigma y}}{\sigma[(\sigma+G\sigma_1)e^{\sigma}-(\sigma-G\sigma_1)e^{-\sigma}]}\times$$

$$e^{i\alpha(\xi-x)}\quad\tau(\xi)\,d\alpha\,d\xi$$

$$w_1(x, y, t) = \mathrm{Re}\,[W_1(x, y)\, e^{-i\omega t}] \tag{1.2}$$

$$W_1(x, y) = \frac{1}{\pi} \int_{-a}^{a} \int_{\Gamma} \frac{\tau(\xi)\, e^{\sigma_1 y}\, e^{i\alpha(\xi - x)}\, d\alpha\, d\xi}{(\sigma + G\sigma_1)\, e^{\sigma} - (\sigma - G\sigma_1)\, e^{-\sigma}}$$

Here

$$\sigma = \sqrt{\alpha^2 - \varepsilon^2 \varkappa_1^2}, \quad \sigma_1 = \sqrt{\alpha^2 - \varkappa_1^2}, \quad \varepsilon = \sqrt{\frac{G\rho}{\rho_1}}, \quad G = \frac{G_0}{G_1}$$

$$x = \frac{X}{h}, \quad y = \frac{Y}{h}, \quad z = \frac{Z}{h}, \quad \alpha = \beta h, \quad \xi = \frac{\Upsilon}{h}, \quad a = \frac{A}{h}$$

The contour Γ coincides with the real axis if the integrand has no zeros and poles on the real axis. Otherwise, the contour Γ deviates from the real axis, passing below the positive singularities of the integrand, as a rule, and above the negative singularities. Setting $y = 1$ in (1.1), we obtain the displacement of the layer surface, which can be written as

$$w_0(x, t) = \mathrm{Re}\,[W_0(x)\, e^{-i\omega t}], \quad W_0(x) = \frac{1}{2\pi} \int_{-a}^{a} k(x - \xi)\, \tau(\xi)\, d\xi \tag{1.3}$$

$$k(x - \xi) = \int_{\Gamma} K(\alpha)\, e^{i\alpha(x - \xi)}\, d\alpha, \quad K(\alpha) = \frac{\sigma \operatorname{ch} \sigma + G\sigma_1 \operatorname{sh} \sigma}{\sigma\,[\sigma \operatorname{sh} \sigma + G\sigma_1 \operatorname{ch} \sigma]}$$

An analysis of the integrand showed that the nature of the wave depends substantially on the relationships between the parameters characterizing the properties of the materials. Waves do not generally originate for certain relationships.

For $\varepsilon < 1$ the integrand in (1.1) evidently has no real roots, i.e. no waves can originate. In this case it can be said that the layer is "stiffer" than the half-space. In the case $\varepsilon > 1$, zeros and poles of the integrand appear on the real axis. Their number increases as ω grows. The behavior of the neutral zeros at once permits indicating the phase velocities and the number of waves originating on the layer surface in this case.

2. In the case of the mixed problem, (1.3) is an integral equation to determine the contact stresses. To solve the integral equation, we introduce the approximation of the function $K(\alpha)$ for $\varkappa_1 = 2$, $\varepsilon = 1.4$. The function $K(\alpha)$ hence has the pole $z_1 = 2.513$ and the zero $\zeta_1 = 2.005$. The approximating function is a polynomial with complex coefficients. The error in the approximation does not exceed 10%, where it is practically zero for $\alpha > 4$. The approximating function can be written as

$$K(\alpha) \approx \frac{H(\alpha)(\alpha^2 - \zeta_1^2)}{\sqrt{\alpha^2 + 100}\,(\alpha^2 - z_1^2)} \tag{2.1}$$

$$H(\alpha) = \frac{(\alpha^2 - \alpha_1^2)(\alpha^2 - \alpha_2^2)(\alpha^2 - \alpha_3^2)(\alpha^2 - \alpha_4^2)}{(\alpha^2 + 25)^4}$$

$$\alpha_1 = 1.838 + 1.149i, \quad \alpha_2 = -0.1173 + 9.754i, \quad \alpha_3 = -3.769 + 6.539i$$

$$\alpha_4 = 3.471 + 6.823i$$

The legitimacy of the approximation can be given a foundation excatly as in [3]. The function $K(\alpha)$ admits of factorization relative to the contour Γ [1]

$$K(\alpha) = K_+(\alpha)\, K_-(\alpha)$$

$$K_\pm(\alpha) = \frac{(\alpha \pm 2.007)(\alpha \pm \alpha_1)(\alpha \pm \alpha_2)(\alpha \pm \alpha_3)(\alpha \pm \alpha_4)}{(\alpha \pm 2.513)\sqrt{10 \mp i\alpha}\,(\alpha \pm 5i)^4} \tag{2.2}$$

Representing the function $K(\alpha)$ in such form, the method in [2] to construct the solution of the integral equation (1.3), as well as to find the displacement of the layer surface outside the stamp, can be used directly.

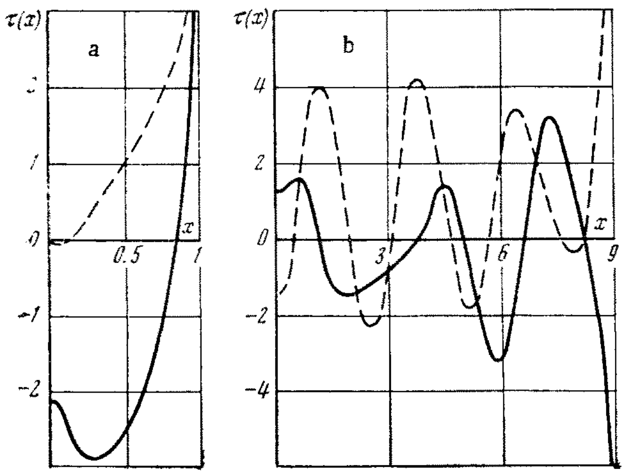

Fig. 2

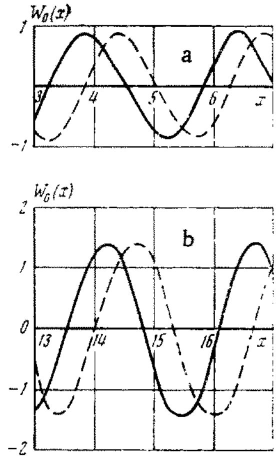

Fig. 3

Without limiting the generality, let us set $W_0(x) = e^{i\eta x}$. The real poles of the function $K_+(\alpha)$ are denoted by $-z_k$, $k = 1, 2, \ldots, m$, and the real zeros by $-\zeta_i$, $i = 1, 2, \ldots, n$. They are all negative. Then the solution of (1.3) can be written in the general case as

$$\tau(x) = -\frac{e^{i\eta x}}{K(\eta)} + \frac{i}{2\pi}\int_{-\infty+ic}^{\infty+ic} \frac{\exp[-iu(a+x) - i\eta a]\,du}{K_+(u)K_+(\eta)(u+\eta)} -$$

$$\frac{i}{2\pi}\int_{-\infty-ic}^{\infty-ic} \frac{\exp[iu(a-x) + i\eta a]\,du}{K_-(u)K_-(\eta)(u+\eta)} + \sum_{k=1}^{n}\Big\langle\Big\{\frac{2i\exp[2ai\zeta_k + i\eta a]}{K_-'(\zeta_k)(\eta+\zeta_k)} +$$

$$\sum_{p=1}^{n} r_k\,[A_{kp}^{+}\alpha^{+}(-\zeta_p) + A_{kp}^{-}\alpha^{-}(-\zeta_p)]\Big\}\Big\{\frac{i\exp[-i\zeta_k(a+x)]}{2K_+(\zeta_k)} +$$

$$\frac{1}{4\pi}\int_{-\infty+ic}^{\infty+ic} \frac{\exp[-iu(a+x)]\,du}{K_+(u)(u-\zeta_k)}\Big\} - \Big\{\frac{2i\exp[2ai\zeta_k - i\eta a]}{K_-'(\zeta_k)(\eta-\zeta_k)} -$$

$$\sum_{p=1}^{n} r_k\,[A_{kp}^{+}\alpha^{+}(-\zeta_p) - A_{kp}^{-}\alpha^{-}(-\zeta_p)]\Big\}\Big\{-\frac{i\exp[-i\zeta_k(a-x)]}{2K_+(\zeta_k)} +$$

$$\frac{1}{4\pi}\int_{-\infty-ic}^{\infty-ic} \frac{\exp[iu(a-x)]\,du}{K_-(u)(u+\zeta_k)}\Big\}\Big\rangle, \quad |x| < a$$

Here $A_{kp}^{\pm}$ are elements of the inverse matrix $\Delta^{\pm}$

$$\Delta^{\pm} = \begin{Vmatrix} 1 \pm \dfrac{K_+(\zeta_1)\, e^{2ai\zeta_1}}{K_+'(-\zeta_1)(\zeta_1+\zeta_1)} & \ldots & \pm \dfrac{K_+(\zeta_n)\, e^{2ai\zeta_n}}{K_+'(-\zeta_n)(\zeta_1+\zeta_n)} \\ \ldots & \ldots & \ldots \\ \pm \dfrac{K_+(\zeta_1)\, e^{2ai\zeta_1}}{K_+'(-\zeta_1)(\zeta_n+\zeta_1)} & \ldots & 1 \pm \dfrac{K_+(\zeta_n)\, e^{2ai\zeta_n}}{K_+'(-\zeta_n)(\zeta_n+\zeta_n)} \end{Vmatrix}$$

$$\alpha^{\pm}(\zeta) = i \sum_{k=1}^{n} \frac{\exp(2ai\zeta_k)}{K_-'(\zeta_k)(\zeta-\zeta_k)} \left[\frac{e^{-i\eta a}}{\zeta_k-\eta} \pm \frac{e^{i\eta a}}{\zeta_k+\eta}\right] -$$

$$\frac{i}{K_-(\zeta)}\left(\frac{e^{i\eta a}}{\zeta-\eta} \pm \frac{e^{-i\eta a}}{\zeta+\eta}\right) + i\left(\frac{e^{i\eta a}}{(\zeta-\eta)K_-(\eta)} \pm \frac{e^{-i\eta a}}{(\zeta+\eta)K_+(\eta)}\right)$$

The expression for $W_0(x)$ has the form

$$W_0(x) \approx \sum_{r=1}^{m} \frac{i}{2\,[K_-^{-1}(z_r)]'} [X^+(z_r) + X^-(z_r)]\, e^{iz_r(x-a)}, \quad x > 2a$$

$$W_0(x) \approx \sum_{r=1}^{m} \frac{i}{2\,[K_-^{-1}(z_r)]'} [X^+(z_r) + X^-(z_r)]\, e^{-iz_r(x-a)}, \quad x < -2a$$

$$X^+(\mp\zeta) \pm X^-(\mp\zeta) = \pm 2i \sum_{k=1}^{n} \frac{\exp(2ai\zeta_k \mp i\eta a)}{K_-'(+\zeta_k)(\zeta_k \pm \zeta)(\eta \mp \zeta_k)} \pm$$

$$2i\,\frac{\exp(\pm i\eta a)}{\zeta+\eta}\left(\frac{1}{K_\pm(\zeta)} - \frac{1}{K_\mp(\eta)}\right) +$$

$$\sum_{k=1}^{n}\sum_{p=1}^{n} \frac{K_+(\zeta_k)\exp(2ai\zeta_k)}{K_+'(-\zeta_k)(\zeta \pm \zeta_k)} [A_{kp}^{\mp}\alpha^{\mp}(-\zeta_p) \mp A_{kp}^{\pm}\alpha^{\pm}(-\zeta_p)]$$

The values of the functions $\tau(x)$ and $W_0(x)$ for the approximation (2.1), (2.2) are computed on an electronic computer for a and η. Presented in Fig. 2 are graphs of the real (solid line) and imaginary (dashed line) parts of $\tau(x)$ for the case of a rigid flat stamp with $a = 1$ and $\eta = 0$ (a) and for $a = 9$ and $\eta = 0$ (b). Presented in Fig. 3 are graphs of the real and imaginary parts of $W_0(x)$ for $a = 1$, and $\eta = 0$ (a) and for $a = 6$ and $\eta = 0$ (b).

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